# DISPERSION AND FIELD ANALYSIS OF A MICROSTRIP MEANDER LINE SLOW-WAVE STRUCTURE

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## Abstract

A structure which is important as an example of a spatially periodic medium for microwave propagation is analyzed theoretically, with rigorous consideration of its partial dielectric composition, translation and reflection symmetry properties, and field configuration relating to its use for electron-wave interaction.

## Introduction

The passage of electromagnetic radiation through spatially periodic media arises in a variety of physical situations, including some as widely diverse as X-ray and particle diffraction by crystals, transmission and reception by microwave array antennas, optical integrated circuits, and radio-frequency filtering. From consideration of the symmetry properties of the medium and the logical consequences of this, it is possible to deduce certain qualitatively distinctive features, such as the existence of stop-bands, properties of the dispersion function, and angular diffraction lobe patterns. For most applications, the generation of useful design information and the realistic comparison of theoretical and experimental results are possible only if the boundary-value problems involved can be solved in a substantially rigorous manner. With the aid of symmetry principles the problems can be reduced to that of field analysis in a single unit cell of the periodic medium. This reduced problem may be itself a rather formidable task, and the value of the results depends on the recognition of significant structural parameters. Various simplified versions of the slow-wave structure for vacuum-tube crossed-field amplifiers have been treated in the microwave literature[1,2].

## Determination of Propagation Parameters

The periodic propagation medium contemplated in the present paper is a printed-circuit configuration to be used as a slow-wave structure in a micrwoave vacuum-tube crossed-field amplifier (CFA). The example presented is that of a meander-line circuit on a dielectric substrate, although it will be seen that the method employed is applicable to a considerable range of slow-wave circuit embodiments, including ladder-lines and interdigital lines among others. The treatment incorporates a number of analytical details which are pertinent to its value for the accurate investigation of the electron-wave interaction on which the action of the amplifier depends. Among these are: the determination of the relative concentration of the microwave field in the dielectric substrate below and the vacuum above the slow-wave circuit, and the consequent dispersive effects; detailed analysis of the microwave electric field in the electron interaction region, leading to determination of the space-harmonic amplitudes and realistic evaluation of an electron-wave interaction impedance. The method permits the use of finite-difference techniques or other numerical analyses of the potential in complicated composite structures, as well as the use of numerically or experimentally determined susceptances associate with structural details such as edges, corners, steps, slots, and bars made of conducting or dielectric materials.

We take the unit cell to contain one complete meander, as indicated in Fig. 1. From the symmetry of the cell and periodicity of the design we can deduce the possible forms of the solutions. Specifically, there must exist basis wave functions, representing normal modes of propagation on the array of Fig. 2, characterized by a phase factor expressing a constant phase increment $\varphi$ per unit cell, and by a factor possessing even or odd symmetry, respectively, with respect to reflection in the central xy-plane of the unit cell. Employing a technique which has previously been used successfully[3] for microstrip analyses, we define a Green's function, representing the potential due to a symmetrical combination of elementary substrips. At this stage the boundary conditions at the conducting ground plane and at the dielectric-vacuum interface are imposed. If the structure includes an upper ground plane (or "sole" in the case of the crossed-field amplifier), such a feature is also represented at this stage. The individual substrips are chosen to be sufficiently narrow that the excitation (visualized as a complex charge or current of unit amplitude) may be taken to be uniform over their cross-sections. In the simplest designs, in which the surface of the dielectric substrate is flat and the conducting meander line is of negligible thickness, the Green's function may be formulated as a Fourier series. We now use this function as a kernel to solve the boundary-value problem for the actual strip configuration. Imposition of the remaining boundary condition for the normal modes, namely that the meander line strips are equipotential, leads to determination of the charge and current profiles on the strips. In this way we obtain the potential and the charge distribution, hence the capacitance, effective dielectric constant (Fig. 3), and characteristic impedance $Z_0$ as functions of $\varphi$, for the normal modes of each symmetry type. A significant new feature in this formulation is the determination of the difference in propagation velocities for normal modes of the two different symmetries. Such an effect is to be expected, of

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course, because of the variation with phase $\varphi$ and with symmetry type of the field distribution in the dielectric and vacuum. This phenonmenon, which has apparently not been treated analytically before, results in the appearance of new stop-band features, in addition to its influence in determining dispersive effects.

Imposition of simple boundary conditions on the voltages and currents on the connecting links at the sides of the meander line leads to the determination of a characteristic equation; viz.,

$$\tan^2 \frac{\varphi}{4} = \left(\frac{Z_{01}}{Z_{02}}\right) \begin{cases} \tan k_1 A/2 \tan k_2 A/2 \\ \cot k_1 A/2 \cot k_2 A/2 \end{cases}$$

where $k_1$ and $k_2$ are the normal-mode propagation constants for the two symmetry types. The solution, namely the dispersion diagram, is shown in Fig. 4. The input impedance (Fig. 5), interaction impedances and other details can then be determined.

Results obtained to date show very good agreement with experimental data obtained at this laboratory, as illustrated by the experimental points shown in Fig. 4.

## The Interaction Impedance

Interaction between the microwave field and an electron beam flowing above the surface is customarily expressed through an interaction impedance, or coupling impedance $Z_{int}$, which measures the strength of the amplifying action (per unit power of the microwave signal). A useful definition[4] of $Z_{int}$ is

$$(Z_{int})_m = \frac{1}{2P\beta^2} \quad \mathrm{Im}\,(E_y\, E_z^*)_m$$

where $E_y$ and $E_z$ are the components of the microwave electric field, P is the microwave power, $\beta$ is the propagation constant for propagation along the z-axis of the slow-wave structure, and m denotes the m-th order spatial harmonic component of the field. Estimates of $Z_{int}$ are usually done relatively crudely, since the space harmonics are rapidly-varying functions of x and y, the beam density distribution is not well known, and the resultant amplification is sensitive to a variety of other parameters of the device anyway. In the present work, the interaction impedance can be determined with unaccustomed accuracy because the fields themselves are accurately known. This permits us to study some of the assumptions often made in this aspect of CFA work and to examine the effects on the strength and character of the interaction of various structural modifications. Examples illustrating the details of these results will be presented. We find that a representative mean value of $Z_{int}$ for m = 0 at the surface is near 50 ohms in the portion of the pass-band contemplated, for the meander line of a CFA design currently under development.

A further capability of the theory is that of treating the dielectric-vacuum and dielectric-metal interfaces accurately even when they involve steps, blocks, slots, etc., as are often required in CFA applications where beam-dynamic and thermal considerations dominate the design. This is done by a method in which the Green's function is determined in two steps. First, a "vacuum" Green's function is generated, in which the dielectric substrate is treated as if its dielectric constant were unity. Second, this function is employed in an iterative process to solve for the kernel of an integral equation expressing the electric displacement boundary condition at the dielectric interface.

## Conclusion

Beyond the immediate objective, namely, to obtain realistic analytical data for experimental comparison and design guidance on a class of crossed-field amplifier circuits, the formulation presented here illustrates how a technique for systematic accounting of the complex fields in periodic media, in any wavelength range of interest, can be carried out by a combination of logical, analytical, and numerical methods.


## Acknowledgment

The interest and encouragement of D. H. Temme and the benefit of stimulating discussions with W. E. Courtney have been very valuable in the development of this work.


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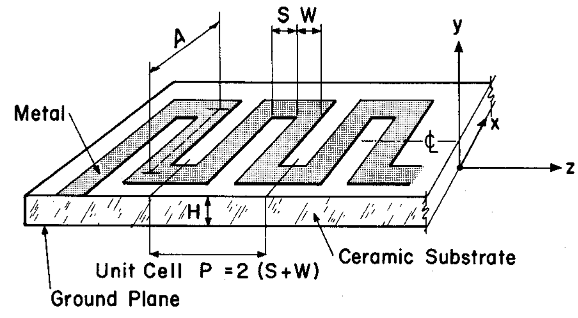


Figure 1. Dimensions and coordinates for the meander line, showing the unit cell.

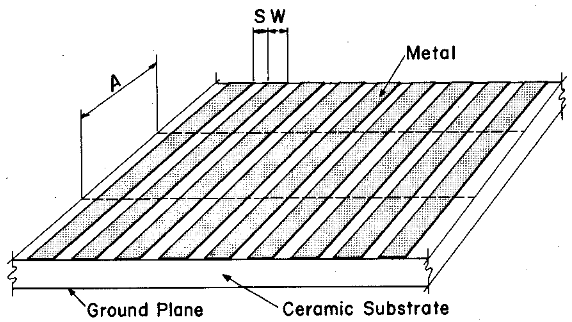


Figure 2. An infinite array of parallel strips, with respect to which the symmetry-adapted normal modes of propagation are defined.

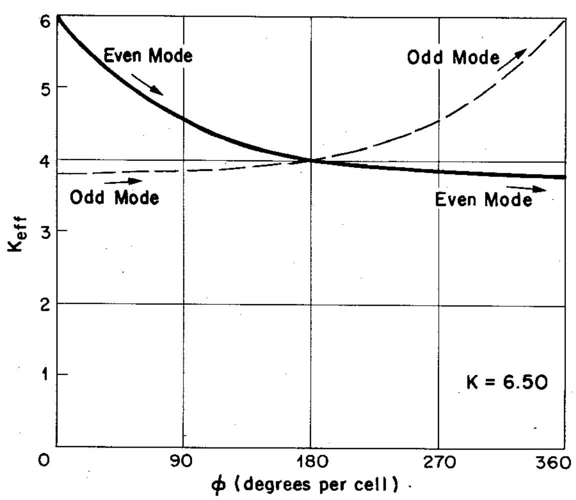


Figure 3. Effective dielectric constant functions $K_{eff}$ of the normal modes.

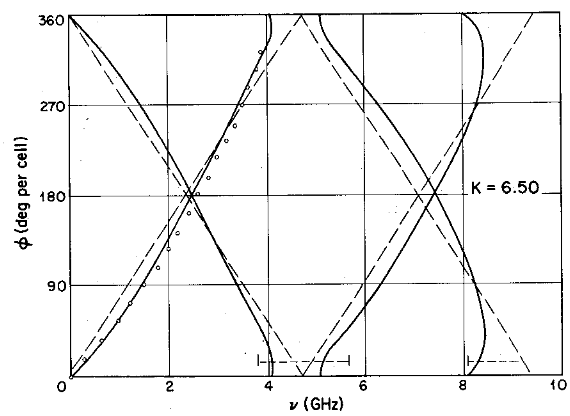


Figure 4. Dispersion diagram for the microstrip meander line. The points and horizontal dashed bars are observed phases and stop bands, respectively. The slanting dashed lines represent the limiting case of uncoupled meander lines.

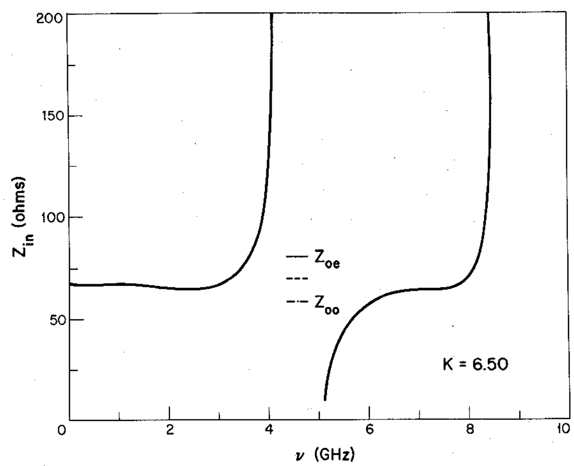


Figure 5. Input impedance function $Z_{in}$ for the microstrip meander line. The three bars in the center of the figure indicate for comparison the characteristic impedances of a coupled pair of lines of the same configuration as that of the meander line unit cell (upper and lower bars) and of a single line (center bar).